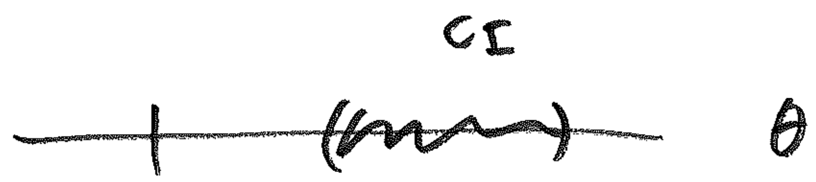
Disc. Sec. week 5

Hospital length of stay (Lecture notes part 5 p. 77) (continued)

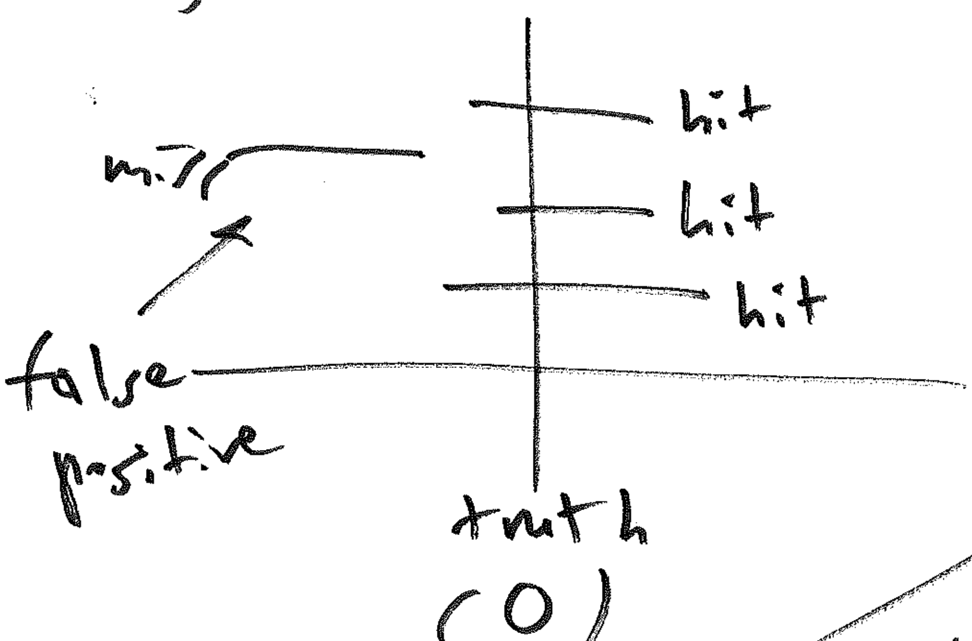
STAT 206 5 Feb 20



0 ↑ my theory incorrect; nothing new

↑ my theory correct; something new

positive +



poisson ↓

$$\ell(\lambda \mid y\, \mathcal{P}\mathcal{B}) = c\, \lambda^{s} e^{-n\lambda}$$

$$p(\lambda \mid \underset{\sim}{Y} \, \mathcal{P}\mathcal{B}) = c\, p(\lambda \mid \mathcal{B}) \left[\ell(\lambda \mid \underset{\sim}{Y} \, \mathcal{P}\mathcal{B}) \right]$$ (2)

$$c\,\lambda^{(\alpha+s)-1} e^{-(\beta+n)\lambda} = \left[c\,\lambda^{\alpha-1} e^{-\beta\lambda} \right] \left[c\,\lambda^{s} e^{-n\lambda} \right]$$

$$\Gamma(\alpha+s,\ \beta+n) \qquad \Gamma(\alpha,\beta) \ (\alpha>0)(\beta>0) \qquad \Gamma(s+1,\ n)$$

$$\begin{pmatrix} \alpha - 1 = s \\ \alpha = s+1 \end{pmatrix}$$

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the Γ family of PDFs (as priors)

is conjugate to the Poisson

sampling model / likelihood

β prior sample size β prior information $I(\alpha, \beta)$

mean $\dfrac{\alpha}{\beta}$

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n data sample size n sample information

mean $\dfrac{s}{n} = \bar{y} = \hat{\lambda}_{MLE}$

$$\begin{pmatrix}\text{posterior} \\ \text{mean}\end{pmatrix} = \begin{pmatrix}\text{prior} \\ \text{mean}\end{pmatrix} \qquad \begin{pmatrix}\text{data} \\ \text{mean}\end{pmatrix}$$

$$\frac{\alpha + s}{\beta + n} = \frac{\beta\left(\dfrac{\alpha}{\beta}\right) + n\left(\dfrac{s}{n}\right)}{\beta + n}$$

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if context suggests a low-information (LI) prior, we want β close to 0 but > 0

here we do want an LI prior, (4)

so let's use $\Gamma(?, \epsilon)$ for

$\epsilon > 0$ but close to 0 ex. $\epsilon = .001$

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popular choice in this model:

$\Gamma(\epsilon, \epsilon)$ prior, ie $\alpha = \beta = \epsilon$

prior mean $E(\lambda \mid \Gamma\, \mathcal{B}) = \frac{\alpha}{\beta} = 1$

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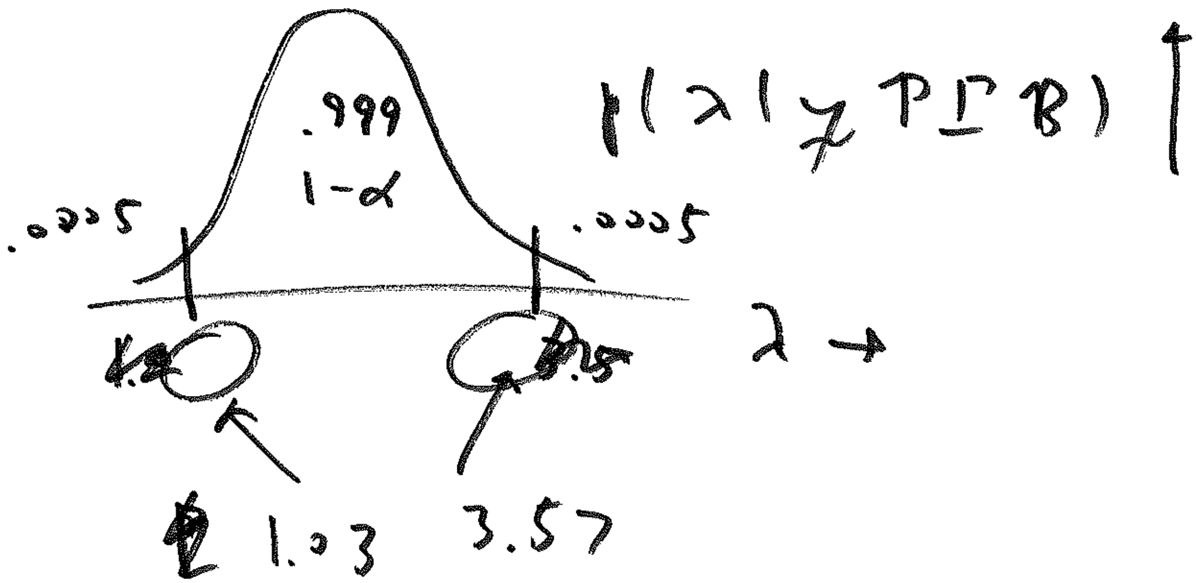
another idea: make $E(\lambda \mid \Gamma\, \mathcal{B}) = \bar{y}$

$\frac{\alpha}{\beta} = \bar{y} = \frac{\alpha}{\epsilon}$, so $\alpha = \bar{y} \cdot \epsilon$ &

you could use $\Gamma(\bar{y} \cdot \epsilon, \epsilon)$ as

your LI prior


p(λ | y P Γ B)
.999
1-α
.0005
.0005
λ →
1.03
3.57