take-home test 1 not due until I've covered all of its material (sometime next week)

\[
p(y_i \mid Z, E, B) = \frac{1}{\lambda} \exp\left(-\frac{y_i}{\lambda}\right) \cdot I(y_i > 0)
\]

for \( y_i > 0 \) for all \( i \)

\[
p(x) \mid Z, E, B = \prod_{i=1}^{n} p(y_i \mid Z, E, B)
\]

\[
= \prod_{i=1}^{n} \frac{1}{\lambda} \exp\left(-\frac{y_i}{\lambda}\right) = \lambda^{-n} \exp\left(-\sum_{i=1}^{n} y_i/\lambda\right)
\]

\[
S = \sum_{i=1}^{n} y_i
\]

\[
\ell(\theta \mid S-1) = -\frac{S}{\theta} - n \ln \theta - n \ln \lambda + \frac{S}{2} - \frac{n}{2}
\]

\[
\theta = \frac{S}{n \lambda} = \frac{S}{n - n \ln \lambda}
\]
\[ p(\theta | \mathbf{x}) = \int p(\mathbf{x}, \theta, \mathbf{y} | \mathbf{z}) \, d\theta \]

\[ \theta = \mathbb{P}(\mathbf{y} = 1 | \theta \mathbf{z}) \]

\[ 0 \leq \theta \leq 1 \]

\[ \mathbb{E} \text{ exchangeable} \]

(Conditionally IID)

Extruding the conversation by many in $\theta$

\[ \int \int \cdots \int \left[ p(\mathbf{x}, \theta) \right] \, d\theta \]

\[ \text{def} \quad p(\mathbf{x} | \theta) \quad \forall \theta (\mathbf{z}) \quad \forall \theta \]

if $\mathbf{z}_1, \mathbf{z}_2, \ldots$

are exh.

\[ p(\mathbf{x} | \theta) \quad \forall \theta (\mathbf{z}) \quad \forall \theta \]

Borell: (6)
\( X \) is prior equivalent to likelihood

\[
(Z_i | Y, \theta) \sim \text{Bernoulli}(\theta) \quad (i = 1, \ldots, n)
\]

Hierarchical model (2 layers)

Posterior PDF

\[
 p(\theta | Y, B) = \frac{p(\theta | B) \cdot p(Y | \theta, B)}{p(Y | B)}
\]

This works

Function of \( \theta \) for fixed \( y \)

\( p(Y | \theta, B) \)

Annoying normalization constant

Must think of sampling dist. as function of \( \theta \)

For fixed \( y \) - this is identical to writing from Laplace & Fisher
\[ I(\alpha) = \frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha + \beta)} = ? \]

\[ I(x) = \int_0^\infty t^{a-1} e^{-t} \, dt \]

\[ I(n+1) = n! \text{ for } n = 0, 1, 2, \ldots \]

\[ (\alpha > 0, \beta > 0) \]

\[ p(\theta | \alpha, \beta, \theta) = c \theta^{a-1} (1-\theta)^{b-1} \]

\[ \text{Beta}(\alpha, \beta) \]

\[ \text{Beta}(1, 1) = \text{Uniform}(0,1) \]
\[ p(\theta | x^B) = c \begin{pmatrix} \theta^{y-1} (1-\theta)^{n-y} \end{pmatrix} \]

\[ s = \sum_{i=1}^{n} x_i \]

\[ y = (y_1, \ldots, y_n) = c \theta^{y+s-1} (1-\theta)^{(n-s)-1} \]

\[ = \text{Beta}(\alpha + s, \beta + n - s) \]

if choice of prior = same

wati form as likelihood

leads to (product of two

such PDFs/PMFs) is another

Raiffa, Schlaifer (1958)

one) that prior is said to

be conjugate to that lik.
Total info about $\theta$

Info internal to $\theta$ and $\beta$

Prior $p(\theta | \alpha, \beta)$

$$E(\theta | \alpha, \beta) = \frac{\alpha}{\alpha + \beta} = 0.15$$

$$\int_{0.05}^{0.3} p(\theta | \alpha, \beta, \beta) d\theta = 0.95$$

$Raith, Schlinker (1958)$