Discussion week 4

Poisson: $(\lambda > 0)$

STAT 206 29 Jan 20

$p_{Y_i}(y_i \mid \lambda \, \mathcal{P} \, \mathcal{B}) =$ ①

$$\begin{cases} \dfrac{\lambda^{y_i} e^{-\lambda}}{y_i!} & y_i = 0, 1, \dots \\ 0 & \text{else} \end{cases}$$

$E(Y_i) = \lambda$

$V(Y_i) = \lambda$

likelihood analysis

① marginal PMF

$\underset{\sim}{Y} = (Y_1, \dots, Y_n)$

$\underset{\sim}{y} = (y_1, \dots, y_n)$

② joint PMF

Poisson

$p_{\underset{\sim}{Y}}(\underset{\sim}{y} \mid \lambda \, \mathcal{P} \, \mathcal{B})$

$(Y_i \mid \lambda \, \mathcal{P} \, \mathcal{B}) \overset{IID}{\sim} \text{Poisson}(\lambda)$

$(i = 1, \dots, n)$

and

$= P(Y_1 = y_1, Y_2 = y_2, \dots, Y_n = y_n \mid \lambda \, \mathcal{P} \, \mathcal{B})$

$$P(A \text{ and } B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$$ general

indep of A, B

$$P(B \mid A) = P(B)$$
$$P(A \mid B) = P(A)$$

$$P(A \text{ and } B) = P(A)\,P(B)$$

(I) ID →

joint = product of marginals

for $y_1, \dots, y_n = 0, 1, \dots$

$$P_{\underset{\sim}{Y}}(\underset{\sim}{y} \mid \lambda P \mathcal{B}) = \prod_{i=1}^{n} P_{Y_i}(y_i \mid \lambda P \mathcal{B})$$

$$= \prod_{i=1}^{n} \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

$$= \left(\frac{\lambda^{y_1} e^{-\lambda}}{y_1!} \right) \left(\frac{\lambda^{y_2} e^{-\lambda}}{y_2!} \right) \cdots \left(\frac{\lambda^{y_n} e^{-\lambda}}{y_n!} \right)$$

3

$$= \frac{\lambda^{y_1 + \cdots + y_n} e^{-n\lambda}}{\prod_{i=1}^{n} y_i!}$$

③ think of joint PMF as fn of λ for fixed $\underset{\sim}{y}$

$s = \sum_{i=1}^{n} y_i$

$$\boxed{\ell(\lambda \mid \underset{\sim}{y} \, \mathcal{P} \, \mathcal{B})} = \frac{\lambda^{s} e^{-n\lambda}}{\prod_{i=1}^{n} y_i!}$$

$$= c \, \lambda^{s} e^{-n\lambda}$$

$(\lambda > 0)$

④ since $\ell(\lambda \mid \underset{\sim}{y} \, \mathcal{P} \, \mathcal{B})$ depends on $\underset{\sim}{y}$ only through $s = \sum_{i=1}^{n} y_i$, s is a sufficient statistic

λ dim 1
s dim 1

strictly speaking need both (s, n) to evaluate lik. f'n (4)

(5) log lik

$\ell\ell(\lambda \mid \mathcal{Y}\, \mathcal{P}\mathcal{B})$

$\ell(\lambda \mid \mathcal{Y}\, \mathcal{P}\mathcal{B}) = \cancel{c}\, \lambda^s e^{-n\lambda}$

$= s \log \lambda - n\lambda$

(6) compute $\hat{\lambda}_{MLE}$:

$$\frac{\partial}{\partial \lambda} \ell\ell(\lambda \mid \mathcal{Y}\, \mathcal{P}\mathcal{B}) = \frac{s}{\lambda} - n = 0$$

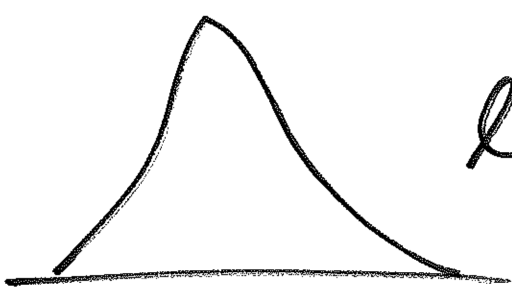
iff $\lambda = \hat{\lambda}_{MLE} = \frac{s}{n} = \bar{y}$ ÷

2.1

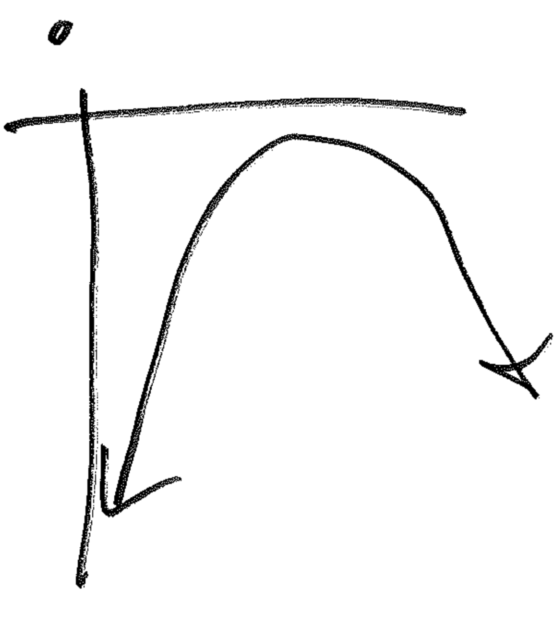
normal density for λ

$$c_1 e^{-c_2(\lambda - c_3)^2} = \ell(\lambda|\sim)$$

$$\ell\ell(\lambda|\sim) = c_4 - c_2(\lambda - c_3)^2$$



$\ell(\lambda|\sim)$



$\ell\ell(\lambda|\sim)$

$\hat{\lambda}_{MLE} = \frac{s}{n} = \bar{y}$

$\ell\ell(\lambda \mid y\, \mathcal{P}\, \mathcal{B}) =$

$s \log \lambda - n \lambda$

inference

(7) standard error of MLE

Neyman: $V_{RS}(\hat{\lambda}_{MLE}) = \frac{\sigma^2}{n}$

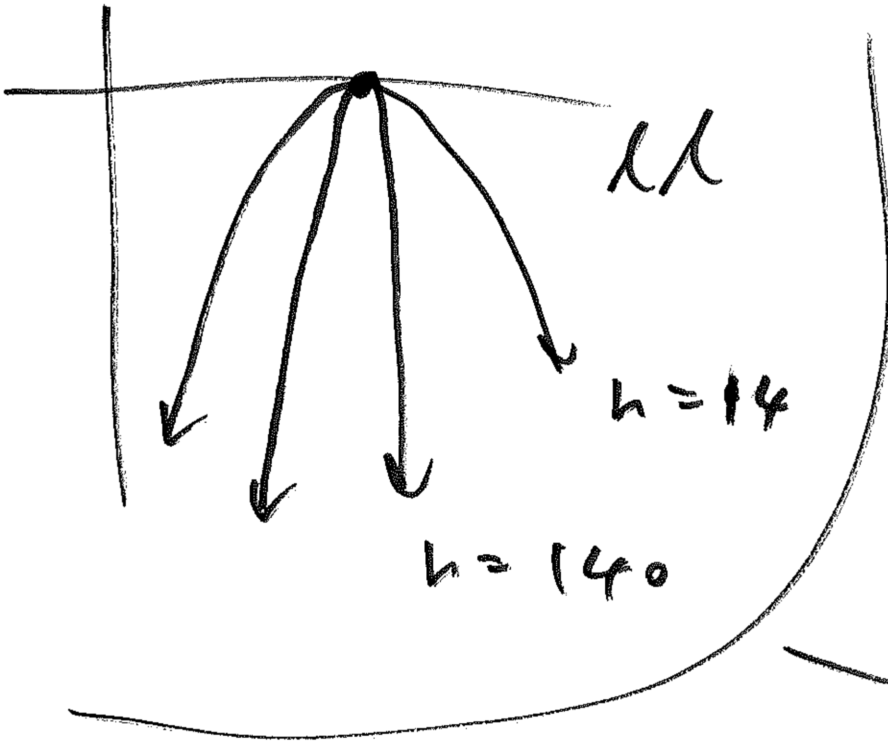
but (Poisson) $\sigma^2 = \lambda$, so

$V_{RS}(\hat{\lambda}_{MLE}) = \frac{\lambda}{n}$

$SE_{RS}(\hat{\lambda}_{MLE}) = \sqrt{\frac{\lambda}{n}}$

$\hat{SE}_{RS}(\hat{\lambda}_{MLE}) = \sqrt{\frac{\hat{\lambda}}{n}}$

Fisher



$$\hat{I}(\hat{\lambda}_{MLE}) = \left[-\frac{\partial^2}{\partial\lambda^2} ll(\lambda|\sim)\right]_{\lambda=\hat{\lambda}_{MLE}}$$

$$ll(\lambda|\sim) = s\log\lambda - n\lambda$$

$$\frac{\partial}{\partial\lambda} ll(\lambda|\sim) = \frac{s}{\lambda} - n$$

$$\frac{\partial^2}{\partial\lambda^2} ll(\lambda|\sim) = -\frac{s}{\lambda^2}$$

$$\left[-\frac{\partial^2}{\partial\lambda^2} ll(\lambda|\sim)\right]_{\lambda=\hat{\lambda}_{MLE}} \overset{*}{=} \frac{s}{\left(\frac{s}{n}\right)^2} = \frac{n^2}{s}$$

$(*) = \frac{n^2}{s} = n \cdot \left(\frac{n}{s}\right) = \frac{n}{\hat{\lambda}_{MLE}}$

$I(\hat{\lambda}_{MLE}) = O(n)$

$V_{RS}(\hat{\lambda}_{MLE}) =$

$\left[I^{-1}(\hat{\lambda}_{MLE}) \right]^{-1} = \frac{\hat{\lambda}_{MLE}}{n}$ ✓

$\hat{SE}_{RS}(\hat{\lambda}_{MLE}) = \sqrt{\frac{\hat{\lambda}_{MLE}}{n}} = 0.38$

(8) CI miss $\overset{\text{often}}{\leftrightarrow}$ false positive

(we claim something new is true but actually our result was just unlucky random sampling

false ⊕ claim

ex. cold fusion when actually not so

worse

false ⊖ : not claiming cold fusion when true

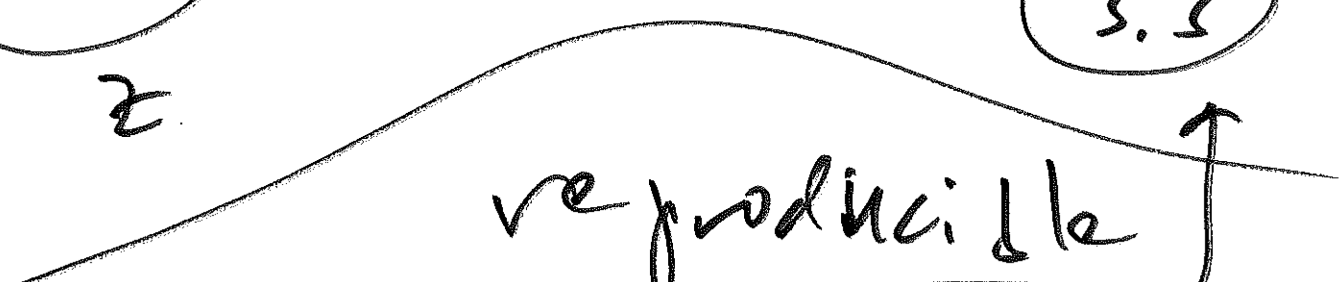
bad but not as bad

~~95%~~ intervals → 99.9% int.

1.96 ≐ 2

z

3.3



reproducible

science

$\hat{\theta}_{MLE} \pm 3.3 \, \hat{SE}_{RS}(\hat{\theta}_{MLE})$