LN pt. 8 p. 155

$V_{RS}(\mathcal{I}_i) = \sigma^2$

parameter space $= [0, \infty)$ if MLE occurs on boundary, ML story harder

$\hat{\gamma}_{MLE} = (\hat{\sigma^2})_{MLE} = \underline{41.3}$

$\eta = \sigma$ would be nice if

$$\hat{\sigma}_{MLE} = \hat{\eta}_{MLE} = \sqrt{\hat{\gamma}_{MLE} = (\hat{\sigma^2})_{MLE}}$$

i.e. would be nice if, for a nice function g, $g(\hat{\theta}_{MLE}) = \widehat{g(\theta)}_{MLE}$

$$(Y_i \mid \mu\,\sigma\,\mathcal{G}\,\mathbb{B}) \stackrel{IID}{\sim} N(\mu, \sigma^2) \quad (i = 1, \dots, n)$$

both unknown

$(\bar{y}, s^2)$ suff. for $\underset{\sim}{\theta} = (\mu, \sigma^2)$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$\hat{\mu}_{MLE} = \bar{y}$$

$$\widehat{(\sigma^2)}_{MLE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

not $\frac{1}{n-1}$

$\boxed{Q:}$ why do freq. like s^2 instead of $\widehat{(\sigma^2)}_{MLE}$?

$\boxed{A:}$ in R.S. s^2 is <u>unbiased</u> for σ^2: $E_{RS}(s^2) = \sigma^2$ $(\$^2)$

(however, it's not true that

$E_{RS}(s) \neq \sigma$ ($)

so this is an example of a general property of MLEs: they can be biased

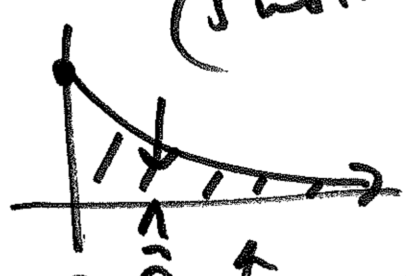
general fact (*) $E_{RS}\left(\hat{\theta}_{MLE}\right) = \theta + \underline{\underline{O}}\left(\frac{1}{n}\right)$

(bias of order $\frac{1}{n}$)

bias

(small sample sizes)

$\ell(\sigma \mid y \ldots B)$ $(\sigma \geq 0)$



$\hat{\sigma}_{MLE} = 0$

MLE is poor summary of ℓ —

$(Y_i \mid \mu\ \sigma\ \mathcal{G}\ \mathcal{B}) \overset{IID}{\sim} N(\mu, \sigma^2)$ unknown

$(i = 1, \ldots, n)$ ← small

1900

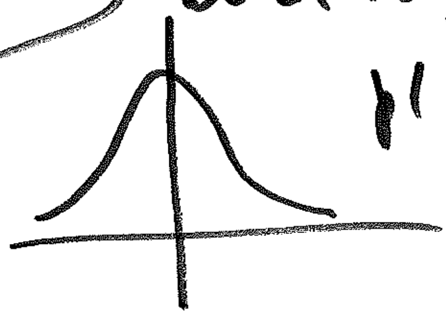
$\bar{y} \pm 1.96 \frac{s}{\sqrt{n}}$ ← not v. accurate for n small

$\bar{y} \pm 1.96 \frac{\sigma}{\sqrt{n}}$ ← known

$\frac{\bar{y} - \mu}{s/\sqrt{n}} \overset{RS}{\sim} t_{n-1}$

Gosset (1908)

(5 2) $\text{Beta}(\underset{\alpha^*}{75.5}, \underset{\beta^*}{353.5})$



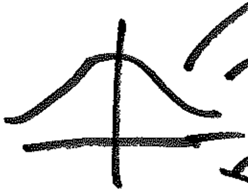
$p(\theta \mid y\ \text{Beta}(\alpha, \beta)\ \mathcal{B})$

$E(\theta \mid y\ \text{Beta}(\alpha, \beta)\ \mathcal{B}) = \theta_{PM}$

$\frac{\alpha^*}{\alpha^* + \beta^*} = 0.1779$

θ^*

IID



0.1779 post. mean

$$\begin{bmatrix} \theta_1^* \\ \theta_2^* \\ \vdots \\ \theta_m^* \end{bmatrix} \quad m$$

Monte Carlo data set

# of monitoring iterations

sample mean $\bar{\theta}^*$

sample SD s_θ

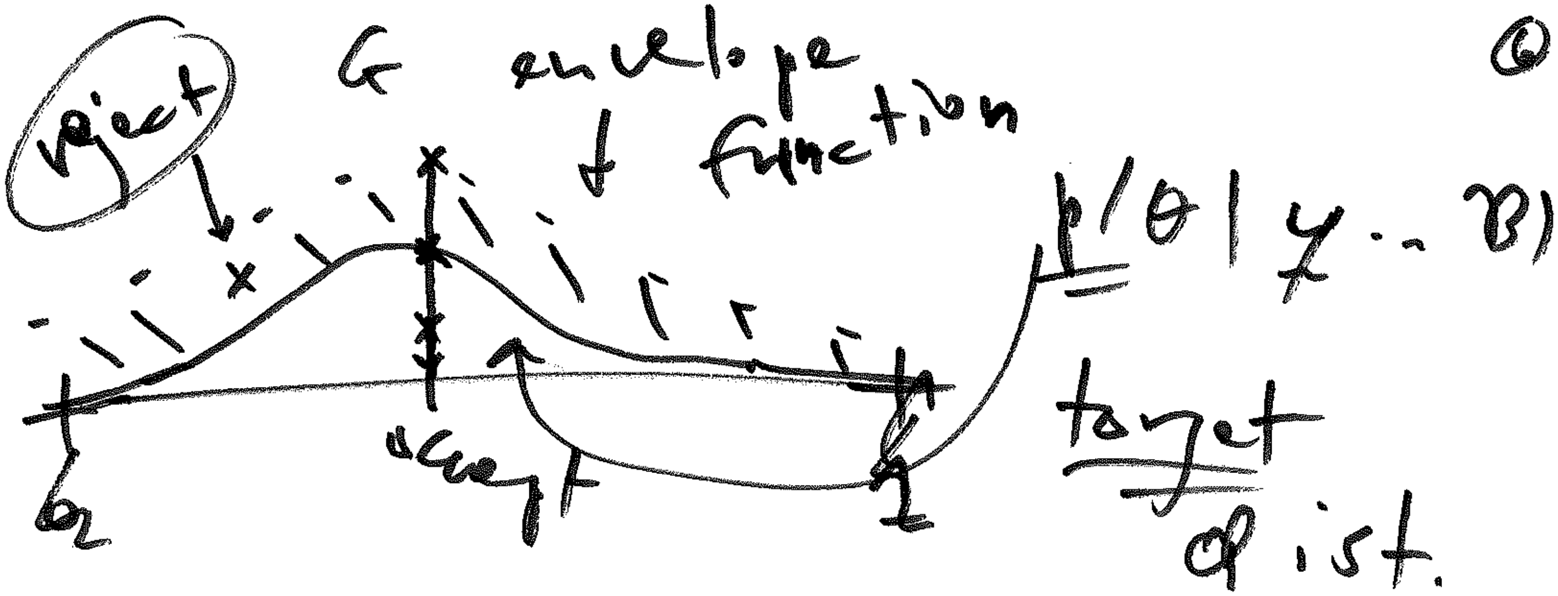
our Monte Carlo est. of post. mean θ_{PM}

$\bar{\theta}^* \xrightarrow{P} \theta_{PM}$ as $m \to \infty$

how accurate is $\bar{\theta}^*$ as a Monte Carlo est. of θ_{PM}?

monte carlo standard error

$$\widehat{MCSE}\left(\bar{\theta}^*\right) = \frac{s_\theta}{\sqrt{m}} \leftarrow = \epsilon$$

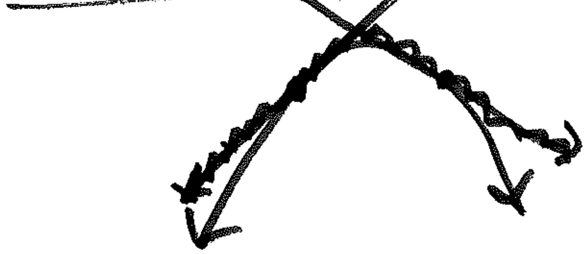


① $G(\theta) \geq p(\theta \mid —) \geq 0$

② $\int G(\theta)\, d\theta < \infty$ → can normalize to get PDF g

③ g easy to sample from

log concave
$p(\theta \mid y \ldots B)$



$\log p(\theta \mid y \ldots B)$