The parameter space is $[0, \infty)$ if MLE occurs on boundary. MLE story harder.

If $\tilde{\eta} = 0$ would be nice if

$$\tilde{\sigma}_{\text{MLE}} = \tilde{\eta}_{\text{MLE}} = \sqrt{\frac{\tilde{\eta}_{\text{MLE}} - \tilde{\eta}}{\text{MLE}}}$$

where

$$\tilde{\eta}_{\text{MLE}} = 41.3$$

If would be nice if, for a nice function $g$, $g(\tilde{\theta}_{\text{MLE}}) - g(\theta_{\text{MLE}})$
\( \theta \sim N(\mu, \sigma^2) \)

\( \bar{y}, \sigma^2 \) suff. for \( \theta = (\mu, \sigma^2) \)

\[
\hat{s}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2
\]

\[
\hat{\sigma}^2_{MLE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2
\]

**Q:** why do freq. like \( s^2 \) instead of \( \hat{\sigma}^2 \) MLE? in r.s. \( s^2 \) is unbiased

for \( \sigma^2 \): \( E_{\theta \sim N(\mu, \sigma^2)}(s^2) = \sigma^2 \) (Q3)
However, it's not true that $E_{\theta}(\hat{\theta}) \neq 0$.

So this is an example of a general property of MLEs: they can be biased.

A general fact:

$E_{\theta}(\hat{\theta}_{\text{MLE}}) = 0 + O(\frac{1}{n})$ (bias of order $\frac{1}{n}$)

This is especially true for small sample sizes (where the MLE is poor).

MLE is heavily skewed.

\[ \hat{\theta}_{\text{MLE}} = 0 \]

Summary of $\mathbf{E}$
\( (X_1, \ldots, X_n) \sim \text{IID } N(\mu, \sigma^2) \)

\[ \mu \sim \text{unknown} \]

\[ \bar{Y} \pm 1.96 \frac{\sigma}{\sqrt{n}} \quad \text{not v. occurr for normal} \]

\[ \bar{Y} \pm 1.96 \frac{\sigma}{\sqrt{n}} \quad \text{known} \]

\[ \bar{Y} - \mu \sim \frac{\sigma}{\sqrt{n}} \text{ t}_n \]

\[ C \quad \text{Beta} \left( 75.5, \frac{1}{2} \right) \]

\[ 1.61 \gamma \text{ Beta} \left( a, \beta \right) \]

\[ E(\theta | \gamma) \text{ Beta}(\alpha, \beta) \]

\[ 0 \text{ or } \gamma \]

\[ \frac{1}{\lambda^2} = 0.1779 \]
IID

\[ \begin{pmatrix}
\theta^* \\
\theta_1^* \\
\vdots \\
\theta_m^*
\end{pmatrix}
\]

Upper bound

Monte Carlo data set

Number of monitoring iterations

Est. of post. mean \( \hat{\theta}_m \)

\[ \hat{\theta} \to \theta_m \ \text{as } m \to \infty \]

How accurate is \( \hat{\theta}^* \) as a Monte Carlo est. of \( \theta \) in

Monte Carlo standard error

\[ \text{MCSE}(\hat{\theta}^*) = \frac{5\theta}{5m} \left(1 - \epsilon\right) \]
\( G(\theta) \geq p(\theta|y \cdots y) \geq 0 \)

1. \( \int G(\theta) d\theta < \infty \) \implies \text{can normalize to get } P(D|y) = g

2. \( g \) easy to sample from

3. \( \text{by concave } \)